

Effects of Sloshing on Analysis and Design of Elevated Service Reservoirs (ESR)

Abhijeet. B. Babar¹ and H. S. Jadhav²

¹PG Student at R.I.T Islampur

²R.I.T Islampur

E-mail: ¹abhijeetbabar3@gmail.com, ²hanmant.jadhav@ritindia.edu

Abstract—Water supply is a life line facility that must remain functional in natural disaster. These structures have large mass concentrated at the top of slender supporting structure hence these structure are especially vulnerable to horizontal forces due to earthquakes. The ESR were collapsed or heavily damaged during the earthquakes because of unsuitable design of supporting system or wrong selection of supporting system and underestimated demand or overestimated strength. So, it is very important to select proper supporting system and also need to study the response of ESR to dynamic forces by both equivalent Static method as well as Dynamic method and to find out the design parameters for seismic analysis. It is also necessary to consider the sloshing effect on container roof slab. The effect of hydrodynamic pressure and pressure due to wall inertia & effect of vertical ground acceleration in the seismic analysis must be considered in the seismic analysis of ESR.

1. INTRODUCTION

All over the world, most of the failures of large water tanks after/during earthquakes are suspected to have resulted from dynamic buckling caused by overturning moments of seismically induced liquid inertia and surface slosh waves and also because of unsuitable design of supporting system or wrong selection of supporting system and underestimated demand or overestimated strength.

The Sloshing is defined as, any motion of the free liquid surface inside the container, caused by any disturbance to a partially filled liquid container. Earthquake excitation causes sloshing of water inside the container of service reservoirs creating additional forces on its walls and roof. During earthquake this effect changes the usual problem of elevated service reservoir from single degree of freedom problem to two degree of freedom problem. It is necessary to consider the sloshing of water inside the container and its effect on the roof slab, floor slab, walls of the container and further on the overall behavior of the structure. It is also important to select proper supporting system and the need to study the response of Elevated Water Tanks to dynamic forces and to find out the design parameters for seismic analysis.

2. THE SCOPE AND OBJECTIVE OF THE PROPOSED WORK

This study is focused on the effects of sloshing on Analysis and Design of Elevated service reservoirs. The objectives of this study are as follows: -

- 1) To study the effect of sloshing of liquid/water on overall behaviour of container.
- 2) To analyze the Elevated and Ground supported service reservoir considering the sloshing effect of liquid (Dynamic Method) so as to obtain the Hydrodynamic and Hydrostatic Pressures on Tank wall and Base slab of the container.
- 3) To analyse the Elevated service reservoir without considering the sloshing effect of liquid (Static Method) and comparing its results with those obtained by using Dynamic Method.
- 4) To consider effect of different staging systems on the overall behaviour of E.S.R.

The comprehensive study will be concluded by suitable conclusions based on the results obtained.

3. REVIEW OF LITERATURE

Much of a literature has presented in the form of technical papers till date on the dynamic analysis of Elevated Water Tanks. Different issues and the points are covered in that analysis i.e. dynamic response to ground motion, sloshing effect on tank, dynamic response of framed staging etc. Some of those are analyzed below

George W. Housner (1963) had studied about the relation between the motion of water in the tank with respect to tank and motion of whole structure with respect to ground. He has considered three basic conditions for this analysis. He studied that if water tank is fully filled i.e. without free board then the sloshing effect of water is neglected, if the tank is empty then no sloshing as water is absent. In above two cases water tower will behave as one-mass structure. But in case if there is free board, the water tower will behave as two-mass structure.

Finally, he concluded that the tank fully filled is compared with the partially filled tank then it is seen that the maximum force to which the half-full tank is subjected may be significantly less than half the force to which the full tank is subjected. The actual forces may be as little as 1/3 of the forces anticipated on the basis of a completely full tank.

Sudhir K. Jain & Sameer U. S. (1991) Revised the IS code provision for seismic design of elevated water tanks. It is seen that, due to absence of a suitable value of performance factor for tanks, the code provision for rather low seismic design force for these structure. Simple expressions are derived, which allow calculations of staging stiffness, and hence the time period, while incorporating beam flexibility. The code must include an appropriate value of performance factor, say 3.0 for calculation of seismic design force for water tanks. An earthquake design criterion is incomplete, unless clear specifications are include on how to calculate the time period. A method for calculating the staging stiffness including beam flexibility and without having to resort to finite element type analysis has been presented. This method is based on well-known portal method which has been suitably developed to incorporate the beam flexibility and the three dimensional behaviour of the staging.

O. R. Jaiswal, Shraddha Kulkarni (2008) In this paper, the author had done experimental and numerical study to obtain the sloshing frequency of liquid contained in tanks of different shapes and tanks with internal obstructions. The experimental study is done on laboratory models of tanks, which are excited using an Electro-Magnetic Shake Table. The numerical study is done with the help of finite element model of tank-fluid system using ANSYS software.

Gaikwad M. V. (2013) From detail study and analysis it was found that, for same capacity, same geometry, same height, with same staging system, with same Importance factor & Response reduction factor, in the same Zone; response by equivalent static method to dynamic method differ considerably. Even if we consider two cases for same capacity of tank, change in geometric features of a container can shows the considerable change in the response of tank. As the capacity increases, difference between the response increases. Increase in the capacity shows that difference between static and dynamic response is in increasing order. It is also found that, for small capacity of tank the impulsive pressure is always greater than the convective pressure, but it is vice-versa for tanks with large capacity. Magnitude of both the pressure is different.

4. THEORETICAL FORMULATIONS AND METHODOLOGY

In this chapter the theoretical formulae's and methodology used for the analysis of Ground supported water tanks and Elevated water tanks are given. These theoretical formulae's and methodology used for the analysis are based on the IITK-

GSDMA Guidelines on Seismic Design of Liquid Storage Tanks" (Provisions with commentary) given by Sudhir K. Jain and O R Jaiswal, in October 2007. The basis of these formulations is to evaluate the Hydrodynamic forces exerted by sloshing liquid on tank wall and tank base slab in addition to hydrostatic forces. Thus in order to include the effect of hydrodynamic pressure in the analysis, the tank is idealized as an equivalent spring mass model as stated by Housner, which includes the effect of tank wall – liquid interaction. The parameters of this model depend on geometry of the tank and its flexibility. The approach followed for the analysis of different water tank problems is same, only the formulation and parameters used in each case vary according to the Geometry and support conditions.

4.1 Parameters of Spring Mass Model

The parameters of spring mass model depend on the tank geometry i.e. on h/D ratio for circular tanks which can be obtained directly from the Fig. 4.1.

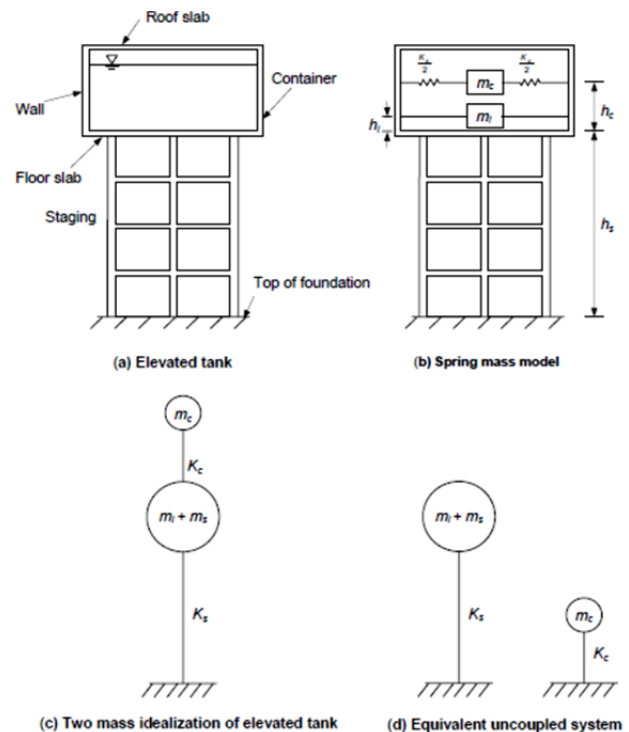


Fig. 1: Two mass idealizations for elevated tank

5. ELEVATE WATER TANK

In this two simple problems of Elevated Water Tanks,

- 1) 50 m³ capacity, supported on four column RC staging system and
- 2) 200 m³ capacity, supported on six column RC staging system are analyzed to obtain Base shear, Base moment, sloshing wave height, Hydrostatic pressure and Hydrodynamic pressures exerted by sloshing liquid on

tank wall and tank base. The analysis is carried out using the formulation of IITK-GSDMA which are based on G. W. Housner’s theory of tank-liquid interaction which is further idealized into spring mass mathematical model.

5.1 Example 1 - Tank Supported on 4 Column RC Staging

Data: A Circular Elevated RC water tank

1. Capacity of 50 m³
2. Inside diameter of tank = 4.65 m
3. Height is 3.0 m (including a free board of 0.3 m)
4. Tank wall and Base slab = 200 mm thick.
5. Grade of concrete is M20. Tank is located on soft soil in seismic zone II.
6. Density of concrete is 25kN/m³.

Table 1- Sizes of Various Components

Component	Size (mm)
Roof Slab	120 thick
Wall	200 thick
Floor Slab	200 thick
Gallery	110 thick
Floor Beams	250 x 600
Braces	300 x 450
Columns	450 dia.

Table 2: Weight Calculations of Various Components

Component	Calculations	Weight (kN)
Roof Slab	$\frac{\pi}{4} * [(5.05)^2 * (0.12 * 25)]$	60.1
Wall	$\Pi * [4.85 * 0.20 * 3.30 * 25]$	251.4
Floor Slab	$\frac{\pi}{4} * [(5.05)^2 * (0.2 * 25)]$	100.2
Gallery	$\frac{\pi}{4} * [(7.05^2) - (5.05^2) * (0.110 * 25)]$	52.3
Floor Beams	$\Pi * [4.85 * 0.25 * (0.60 - 0.20) * 25]$	38.1
Braces	$3.43 * 0.30 * 0.45 * 4 * 25$	186.1
Columns	$\frac{\pi}{4} * [(0.45^2) * 11.7 * 4 * 25]$	185.5
Water	$\frac{\pi}{4} [(4.65^2) * 3 * 9.81]$	499.8

5.1.1 Weight Calculations and Centre of Gravity

- Weight of staging = 186.1 + 185.2 = 371.3 kN.
- Weight of empty container = 60.1 + 251.4 + 100.2 + 38.1 + 52.3 = 502.1 kN.
- Hence, wt. of container + one third wt. of staging = 502.1 + 371.3 / 3 = 626 kN.
- Mass of empty container + one third mass of staging

$ms = (502.1 + 371.3 / 3) * (1,000 / 9.81) = 63,799 \text{ kg.}$

- Centre of gravity of empty container

5.1.2 Parameters of Spring Mass Model

Weight of water = 499.8 kN = 4, 99,800 N.

Hence, mass of water, $m_w = 4, 99,800 / 9.81 = 50,948 \text{ kg.}$

Expression for Parameters of Spring Mass Model

For, $h/D = 3.0 / 4.65 = 0.65,$

$m_i / m = 0.65; m_i = 0.65 * 50,948 = 33,116 \text{ kg.}$

$m_c / m = 0.35; m_c = 0.35 * 50,948 = 17,832 \text{ kg}$

$h_i / h = 0.375; h_i = 0.375 * 3.0 = 1.13 \text{ m}$

$h_i^* / h = 0.64; h_i^* = 0.64 * 3.0 = 1.92 \text{ m}$

$h_c / h = 0.65; h_c = 0.65 * 3.0 = 1.95$

$h_c^* / h = 0.73; h_c^* = 0.73 * 3.0 = 2.19 \text{ m.}$

Table 3 - Obtained Parameters after Analysis

Sr. No	Impulsive mode	Convective mode
1	Time period $T_i = 0.80 \text{ sec}$	$T_c = 2.26 \text{ sec}; (C_c = 3.29)$
2	Design Horizontal Seismic Coefficient $A_{hi} = 0.06$	$A_{hc} = 0.04$ $Z = 0.1, I = 1.5, R = 2.5, (S_a/g)_i = 1.97/1.68 = 0.994 \text{ (Impulsive)}$ $(S_a/g)_c = (0.562 * 1.75) = 0.983 \text{ (Convective)}$
3	Base Shear $V_i = 59.9 \text{ kN}$	$V_c = 7.0 \text{ kN}$
4	Total Base Shear, $V = 60 \text{ kN}$	
5	Base Moments i.e. Overturning Moment at the Base of Staging $M_i^* = 924 \text{ kNm}$	$M_c^* = 113 \text{ kNm}$ Total Moment $M^* = 931 \text{ kNm}$

Table 4 - Hydrodynamic Pressure Calculations

Sr. No	Impulsive mode	Convective mode
1	Hydrodynamic Pressure Calculations $(P_{iw})_{y=0} = 1.41 \text{ kN/m}^2$	$(P_{cw})_{y=0} = 0.12 \text{ kN/m}^2$ $P_{ib} = 0.95 \text{ kN/m}^2$ $(P_{cw})_{y=h} = 0.69 \text{ kN/m}^2$ $(P_{cb})_{y=0} = 0.13 \text{ kN/m}^2$
2	Pressure Due to Inertia of Wall $P_{ww} = 0.32 \text{ kN/m}^2$	
3	Pressure Due to Vertical Excitation $P_v = 1.47 \text{ kN/m}^2$	
4	Maximum Hydrodynamic Pressure, $P = 2.27 \text{ kN/m}^2$	
5	Hydrostatic Pressure, $\rho * g * h = 29.43 \text{ kN/m}^2$	
Therefore P is Found to be 8% of Hydrostatic Pressure.		

5.1.3 Sloshing Wave Height

Maximum sloshing wave height,

$$d_{max} = (A_h)_c R * \frac{D}{2} = 0.04 \times 2.5 * \frac{D}{2} = \mathbf{0.23 \text{ m.}}$$

Height of sloshing wave is less than free board of 0.3 m.

5.2 Example 1 - Tank Supported on 6 Column RC Staging

Data: Circular Elevated RC water tank

1. Capacity of 200 m³
2. Inside diameter of tank = 8.0 m
3. Height is 4.0 m (including a free board of 0.3 m)
4. Tank wall thickness = 200 mm
5. Base slab = 360 mm thick
6. Grade of concrete is M25 Tank is located on soft soil in seismic zone-II
7. Density of concrete is 25kN/m³.

Table 5- Sizes of Various Components

Sr. No.	Component	Size (mm)
1	Roof Slab	140 thick
2	Roof Beam	230 x 600
3	Wall	200 thick
4	Floor Slab	360 thick
5	Floor Beam	350 x 700
6	Gallery, 1.2m wide.	120 thick,
7	Braces	300 x 450
8	Internal Braces at Base Level	250 x 300
9	Column, 6 Nos.	350 dia.

Table 6.- Weight Calculations of Various Components

Component	Calculations	Weight (kN)
Roof Slab	$\Pi \times 0.140 \times 25 [8 + (0.2)^2] / 4$	193.93
Roof Beam	$\Pi \times 0.230 \times 8.4 \times 25 [0.6 - 0.36] / 4$	91.04
Wall	$\Pi \times 4 \times 0.2 \times 25 \times 8.2$	515.22
Floor Slab	$\Pi \times 0.360 \times 25 [8 + 2(0.2)^2] / 4$	498.70
Floor Beam	$\Pi \times 25 \times 0.35 \times 0.7 \times [8 + 0.35]$	160.67
Gallery, 1.2m wide.	$\Pi \times 0.120 \times 25 [10.9^2 - 8.2^2] / 4$	121.50
Braces	$0.450 \times 0.300 \times 25 \times 3 \times 6 \times 3.4$	208.37
Internal Braces at Base Level	$0.250 \times 0.300 \times 25 \times 6 \times 3.4 + 67.635$	106.215
Level	67.635	
Column, 6 Nos.	$\Pi \times 0.35^2 \times 6 \times 25 \times 11.25 / 4$	162.33

Table 6- Spring Mass Model Parameters

For h/D ratio of :- 3.7/8 = 0.4625	
Impulsive Mode	Convective Mode
$m_i = 92979.10\text{kg}$	$m_c = 89259.23 \text{ kg}$
$h_i = 1.387 \text{ m}$	$h_c = 2.29 \text{ m}$
$h_i^* = 3.014 \text{ m}$	$h_c^* = 2.96 \text{ m}$
$\text{Mass of Water } m_w = 185958.20 \text{ kg}$	
$\text{Mass of Container + 1/3 Mass of Staging} = 177265.20 \text{ kg.}$	

Table 7 - Obtained Parameters after Analysis

Sr. No	Impulsive mode	Convective mode
1	Time period	
	$T_i = 1.61 \text{ sec}$	$T_c = 2.98 \text{ sec}; (C_c = 3.29)$
2	Design Horizontal Seismic Coefficient	
	$A_{hi} = 0.0311$	$A_{hc} = 0.268$
	$Z = 0.1, I = 1.5, R = 2.5, (S_a/g)_i = 1.97/1.68 = 0.994 \text{ (Impulsive)}$	
	$(S_a/g)_c = (0.562 * 1.75) = 0.983 \text{ (Convective)}$	
3	Base Shear	
	$V_i = 82.44910 \text{ kN}$	$V_c = 25.743 \text{ kN}$
4	Total Base Shear, V = 86.37 kN	
5	Base Moments i.e. Overturning Moment at the Base of Staging	
	$M_i^* = 13860.58 \text{ kNm}$	$M_c^* = 460 \text{ kNm}$
	Total Moment M* = 1467.21 kNm	

Table 8- Hydrodynamic Pressure Calculations

Sr. No	Impulsive mode	Convective mode
1	Hydrodynamic Pressure Calculations	
	$(P_{iw})_{y=0} = 0.932 \text{ kN/m}^2$	$(P_{cw})_{y=0} = 0.308 \text{ kN/m}^2$
	$P_{ib} = 0.717 \text{ kN/m}^2$	$(P_{cw})_{y=h} = 0.872 \text{ kN/m}^2$
		$(P_{cb})_{y=0} = 0.303 \text{ kN/m}^2$
2	Pressure Due to Inertia of Wall	
	$P_{ww} = 1.525 \text{ kN/m}^2$	
3	Pressure Due to Vertical Excitation	
	$P_v = 1.814 \text{ kN/m}^2$	
4	Maximum Hydrodynamic Pressure, P = 3.069 kN/m²	
5	Hydrostatic Pressure, $\rho * g * h = 36.29 \text{ kN/m}^2$	
	Therefore P is Found to be 8% of Hydrostatic Pressure.	

5.2.1 Sloshing Wave Height

$$d_{max} = A_{hc} * R * D/2$$

$$= 0.295 * 2.5 * 4 = 0.29 \text{ m. (} d_{max} < 0.3 \text{ m Hence ok)}$$

6. COMPARISON BETWEEN DYNAMIC METHOD AND STATIC METHOD

Here the problems analyzed in chapter 5 one of 50 m³ capacity and other of 200 m³ capacity, are analyzed using Static method and further a comparison is made between the results obtained by Dynamic method and Static method.

6.1 Example 1 (Capacity = 50 m³)

6.1.1 Time Period

$$T = 2\pi \sqrt{\frac{m_s + m_s}{K_s}} = 2\pi \sqrt{\frac{50948 + 63799}{6060000}} = 0.864 \text{ sec}$$

6.1.2 Design Horizontal Seismic Coefficient

$$A_{hi} = \frac{Z}{2} * \frac{I}{R} * \left(\frac{S_a}{g}\right) = \frac{0.1}{2} * \frac{1.5}{2.5} * 1.93 = 0.0579$$

6.1.3 Base share

Base shear at the bottom of staging, $V = (A_{hi}) (m + m_s) g$

$$= 65.176 \text{ kN}$$

6.1.3 Base moment

Overturning moment at the base of staging,

$$M = (A_{hi}) [m (h^* + h_s) + m_s h_{cg}] g = 1010.78 \text{ kN-m.}$$

6.2 Example 2 (Capacity = 200 m³)

6.2.1 Time Period

$$= 2\pi \sqrt{\frac{m_s + m_s}{K_s}} = 2\pi \sqrt{\frac{185958.20 + 177265.20}{4089970}} = 1.87 \text{ sec}$$

6.2.2 Design Horizontal Seismic Coefficient

$$A_{hi} = \frac{Z}{2} * \frac{I}{R} * \left(\frac{S_a}{g}\right) = \frac{0.1}{2} * \frac{1.5}{2.5} * 0.893 = 0.0268$$

6.2.3 Base share

Base shear at the bottom of staging $V = (A_{hi}) (m + m_s) g$

$$= 95.45 \text{ kN}$$

6.1.3 Base moment

Overturning moment at the base of staging

$$M = (A_{hi}) [m (h^* + h_s) + m_s h_{cg}] g = 1631.52 \text{ kN-m.}$$

Table 9: Comparison between results of SDOFS and 2DOFS

Problem	Degrees of freedom	Base shear (kn)	Base moment (kn-m)
1	50 m ³	60	931
	SDOFS	65.176	1010.78
2	200 m ³	86.37	1467.21
	SDOFS	95.45	1631.52

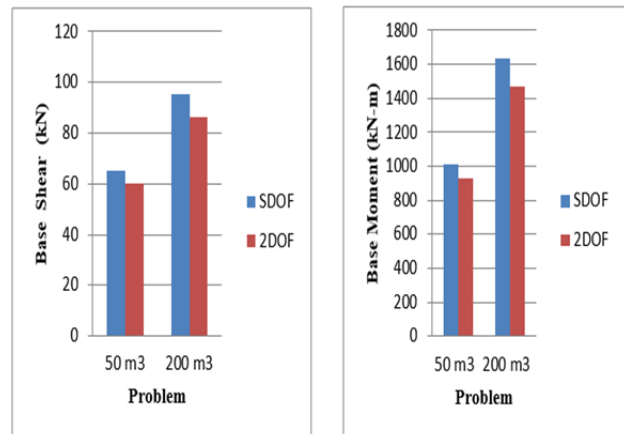


Fig. 2: Comparison between the Base Shears and Base Moments obtained by Static and Dynamic method respectively, for Example 1 and 2

7. CONCLUSION

1. The convective hydrodynamic pressure on base slab and along the height of tank wall is almost same and gradually increases as the water level in tank goes on decreasing. The impulsive hydrodynamic pressure on wall as well as on base slab decreases gradually as the water level in tank goes on decreasing.

- a) Thus the effect of convective and impulsive hydrodynamic pressure depends on the level of water in tank, earthquake force to which it is subjected and height of water tank.
- b) The impulsive pressure is proportional to mass of water in tank.

2. While in the case of circular ground supported tanks the maximum pressure is 25.68% of hydrostatic pressure which is less than 33%, hence hydrodynamic pressure in this case does not affect container design

3. Due to complexity of pressure calculations on tank wall and tank base slab and due to dynamical action, suitable charts are necessary to calculate pressures, based on h/D ratios for the design offices.

4. The results obtained from the comparison made between the Static (considering SDOFS) and Dynamic analysis (Considering 2DOFS), it can be concluded that the results obtained from the dynamic analysis for base shear and base moment (for the same problems considered for static analysis keeping the conditions identical) are less as compared to those obtained from static analysis, which is the point of consideration for any engineer. Here the sloshing water when considered reduces the base shear and base moment making the design economical.

This confirms the need to adopt this new approach of analyzing and designing the water tank by considering it as Two degree of freedom system to make its design economical and safe against additional hydrodynamic pressures due sloshing of water.

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